J Mar Sci Technol (2017) 22:426–431
DOI 10.1007/s00773-016-0425-7

ORIGINAL ARTICLE

Model predictive controller design for ship dynamic positioning system based on state-space equations

Wenhua Li[1] · Yuqing Sun[1] · Haiquan Chen[1] · Gang Wang[1]

Received: 15 March 2016 / Accepted: 27 November 2016 / Published online: 2 December 2016

Abstract Model predictive control (MPC) algorithm based on state-space equations was applied to ship dynamic positioning control system. A state estimator was designed to solve the problem that not all the states used can be measured to improve the control accuracy of the system. Through simulation in MATLAB®, this paper analyzed and compared the model predictive controller with or without constraints and the state estimator. Simulation results on a supply ship verify the effectiveness of this proposed model predictive control algorithm based on state-space equations and show that the MPC controller with the state estimator can improve the control effect of dynamic positioning system of ships.

Keywords Ship · Dynamic positioning system · Model predictive control algorithm · State-space equations · The state estimator

1 Introduction

The dynamic positioning (DP) system is a controlled system that can automatically control the thrusters to keep the ship in the desired position or heading or make the ship to track desired trajectories. The control system is the core of the DP system, whose functioning depending critically on its control algorithm. Along with the advancement of control theory, the DP technology has gone through rapid development, and three generations of products have been introduced to the market. The first generation of DP systems used the PID control algorithm, which succeeded in controlling the three degree of freedom in the horizontal plane. But it had inevitable shortcomings such as the difficulties in selecting PID control coefficient, the limitations imposed by varying linear models. The second generation of DP systems combined the optimal control with the Kalman filter to solve the problem of phase lag, and that led to substantial improvement in performance and extension in function, see for example [1] and [2]. However, some shortcomings, such as the requirement for accurate models or large amount of calculation workload had limited the full potential of DP systems. With the rapid development of computing, sensing, and communication technologies, more complex control strategies and theories were proposed. DP systems have been developed from autocontrol into intelligent control as the third generation of DP products was introduced. For example, backstepping algorithms are designed in [3, 4], a nonlinear programming algorithm is tested in [5, 6], passivity-based techniques and advanced control theory are used in [7–9], sliding mode observer [10, 11], output feedback control [12, 13], inverse linear matrix inequality method [14], robust control [15–17] have all been explored. As mentioned in these references, the positioning accuracy and energy consumption have really been improved.

Model predictive control (MPC) is an advanced control methodology that has been widely accepted in industry. Its capability in dealing with constraints for nonlinear and multi-input–multi-output systems makes it very appealing. By incorporating a model and using it to forecast the future response, it allows the control to predict and correct. It has been widely studied in recent years. In 2001, Konsberg released the Green DP system which used MPC. In

✉ Wenhua Li
liwenhua992@aliyun.com

1 Department of Marine Engineering, Dalian Maritime University, 1# Linghai Road, Dalian 116026, Liaoning, China

[18, 19], MPC was applied to DP system and the results were satisfactory. In [20], the constraints existed in DP were taken into account. However, the aforementioned work assumed that all the states used in the DP control system can be measured. To deal with the problems that not all states are measured and noises exist in the measurements, this paper proposes a model predictive controller with a state estimator to improve the control performance of the system. Simulations in Matlab are used to test and verify the effectiveness of the control algorithm.

2 Mathematical model of the ship

Under the marine environment disturbance, surface vessels will generally have six degrees of freedom (DOF) (surge, sway, heave, pitch, roll, yaw). For DP vessels, we usually focus on horizontal motions that means we need to control surge, sway, and yaw motions. To describe the motion of vessels, two coordinate systems are used: north-east-down (NED) frame $O_E - X_E Y_E Z_E$ and body frame coordinate $o_b - x_b y_b z_b$, see Fig. 1. The position and heading vector of the vessel in the NED frame is $\eta = [x, y, \psi]^T$ and the velocity vector in the body frame is $v = [u, v, r]^T$. Therefore, the horizontal kinematic equations of the vessel can be written as

$$\dot{\eta} = J(\psi)v, \tag{1}$$

the rotation matrix $J(\psi)$ is given by

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

For convenience, it is assumed that the DP vessel is operating at low-speed, and the DP thrusters only respond to low-frequency motions. According to [21], the generalized equation of motion for DP vessel can be expressed as:

$$M\dot{v} + Dv = \tau_E + \tau_T, \tag{3}$$

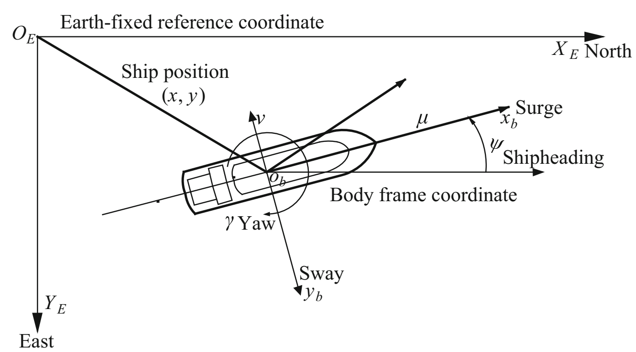

Fig. 1 The NED frame and body frame coordinate system

where τ_E is the vector of environment forces and moments; τ_T is the thrust of DP thrusters; M is the inertia matrix and D is damping matrix.

To avoid the complicated calculation and simplify the design of controller, the linear low-frequency state-space model of the DP vessel using small-angle theory is defined as:

$$\begin{aligned} \dot{X}_L &= A_L X_L + B_L \tau + G_L \omega_L \\ y_L &= C_L X_L + v_L \end{aligned}, \tag{4}$$

where the state vector $X_L = [\eta^T \quad v^T]^T$; τ is the forces and moments of surge and sway; ω_L is unmodeled three-dimensional disturbance vector including wind, wave, and current; y_L is position and heading observation vector (output vector); v_L is the measurement Gaussian white noise. The coefficient matrices in (4) are defined as

$$\begin{aligned} A_L &= \begin{bmatrix} 0_{3\times 3} & I_{3\times 3} \\ 0_{3\times 3} & -M^{-1}D \end{bmatrix}; \; B_L = \begin{bmatrix} 0_{3\times 3} \\ M^{-1} \end{bmatrix}; \\ G_L &= \begin{bmatrix} 0_{3\times 3} \\ M^{-1} \end{bmatrix}; \; C_L = [I_{3\times 3} \quad 0_{3\times 3}]. \end{aligned} \tag{5}$$

3 Algorithm of MPC

The model predictive control is a control algorithm based on model; the basic elements contain predictive model (internal model), receding optimization, and feedback correction. The basic principle can be described as follows: when the new measurement information is obtained at each sampling time, the information will be used to solve an open loop optimization problem defined in a prediction horizon to yield a control sequence whose first component will be used to act on the system, then the process is repeated as new measurement becomes available. The basic structure of MPC is shown in Fig. 2.

While model predictive control is applied to ship dynamic positioning system, first the low-frequency motion information should be separated from comprehensive position information of the vessel, then to control according to the information. The linear low-frequency state-space model of the DP vessel (see Eq. 4) should be converted into discrete state equation, as follows:

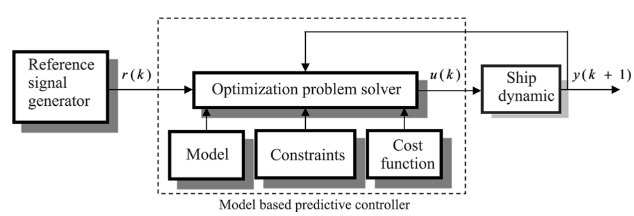

Fig. 2 The basic structure of model predictive control

$$\begin{aligned} x_{k+1} &= A\,x_k + B\,u_k + G\omega_k \\ y_k &= C\,x_k + v_k \end{aligned} \tag{6}$$

the initial state $x_0 = x$; x_k is the state variables' vector about positions and velocities (surge, sway, and yaw) at time k. u_k is the control input vector; ω_k is three-dimensional unmodeled disturbance vector; y_k is the output position vector; v_k is assumed to be Gaussian white noise; A, B, C, G is the constant matrix.

A prediction model for predicting the behavior of the ship in time domain N (sampling period) is established, and the initial conditions are given.

$$\begin{aligned} &x_0 = x \\ &x_{k+1} = Ax_k + Bu_k + G\omega_k, \quad k = 0, \ldots, N-1 \end{aligned} \tag{7}$$

In practical applications, the input and output constraints must be taken into account in design of MPC. The discussion on constrained DP problem can be found in [20, 22]. This paper will take thruster constraints, working area constraints, and operational area constraints into account, weighted linear inequalities is used to represent the operational area, and weighted quadratic inequalities are used to represent the working area, as follows:

$$\begin{aligned} &u_{\min} \le u_k \le u_{\max}; \quad k = 0, 1, \ldots, N-1 \\ &y_k = C_k x_k + v_k \\ &y_k \in \Omega_k; \quad k = 0, 1, \ldots, N-1 \end{aligned} \tag{8}$$

Assuming all states of the system can be measured, the basic MPC algorithm of DP vessels is as follows:

1. Establish the predictive model and set the reference trajectory;
2. Initialization: set the prediction horizon p and control horizon m, $u(-1) = 0$, $x(-1) = 0$;
3. Measure the state value: at time $k \ge 0$, measure x_k, ω_k and v_k, calculate η_k and $\Delta x_k = x_k - x_{k-1}$, then predict the output within future p step and calculate the error $G_1(k+1|k)$ and $u(k) = u(k-1) + \Delta u(k)$;
4. Define the performance indices with constrains: set the above input and output constrains which will be added into the performance indices, so the predictive output can track the reference trajectory accurately.
5. Receding optimization: the finite horizon optimization problem will be solved on line to make the defined performance indices minimum. At each sampling time k, repeat the following steps: (a) the state x_k is obtained from the reference system and the sensor, and is the initial condition of the open loop optimization problem: $x_0 = x_k$; (b) using x_0 to solve the open loop finite time domain optimization problem, the performance index is minimized, and the optimal control sequence is obtained: $U_N^{\mathrm{OPT}} = \{u_0^{\mathrm{OPT}}, \ldots, u_{N-1}^{\mathrm{OPT}}\}$; (c) the first component of the sequence will act on system: $u_k = u_0^{\mathrm{OPT}}$.
6. At the subsequent each sampling time, the algorithm will return (3) to be repeated until the errors between output and set point are minimized or within the allowable range.

4 The design of state estimator

As stated above, to implement the MPC algorithm for a DP system, there exists an essential condition that full state information must be known. However, not all the states of DP system can be measured or disturbance and measurement noise exists. Therefore, to achieve better control precision, a state estimator must be designed to estimate the state variables on line and then the state estimated will be used as the input of the MPC controller.

To design the state estimator, first the measurable output variable equation should be defined as: $y(k) = Cx(k) + v(k)$, the measurement value $y(k)$ is the position (surge, sway) and heading (yaw).

Then design the state estimator as follows:

$$\begin{aligned} &\hat{x}(k+1|k) = A\hat{x}(k|k-1) + Bu(k) + K(k)[y(k) - \hat{y}(k|k-1)] \\ &\hat{y}(k|k-1) = C\hat{x}(k|k-1), \end{aligned} \tag{9}$$

where the first equation is state estimation equation; the second equation is the optimal linear prediction of the measurable output $y(k)$.

(7) and (9) make a difference to get the estimation error $\tilde{x}(k+1|k)$:

$$\begin{aligned} \tilde{x}(k+1|k) &= x(k+1|k) - \hat{x}(k+1|k) \\ &= Ax_k + Bu_k + G\omega_k \\ &\quad - \{A\hat{x}(k|k-1) + Bu(k) + K(k)[y(k) - C\hat{x}(k|k-1)]\} \\ &= [A - K(k)C]\tilde{x}(k|k-1) + G\omega_k + K(k)v(k). \end{aligned} \tag{10}$$

Using orthogonality theorem: $E[\tilde{x}(k+1|k)y^T(k)] = 0$.

$$\begin{aligned} &E\{\{[A - K(k)C]\tilde{x}(k|k-1) + G\omega(k) + K(k)\upsilon(k)\}y^T(k)\} \\ &\quad = E\left\{ \begin{array}{l} \{[A - K(k)C]\tilde{x}(k|k-1) + G\omega(k) + K(k)\upsilon(k)\} \\ \qquad \times\{C[\hat{x}(k|k-1) + \tilde{x}(k|k-1)] + \upsilon(k)\}^T \end{array} \right\} \end{aligned} \tag{11}$$

As $E\{\tilde{x}(k|k-1)\hat{x}^T(k|k-1)\} = 0$; $\upsilon(k)$, $\omega(k)$ and $\tilde{x}(k|k-1)$ orthogonal; $\upsilon(k)$, $\omega(k)$ are white noise with zero mean; so we get the optimal gain matrix $K(k)$:

$$K(k) = [AP(k|k-1)C^T + GS_k][CP(k|k-1)C^T + R_k]^{-1}, \tag{12}$$

where $P(k|k-1) = E[\tilde{x}(k|k-1)\tilde{x}^T(k|k-1)]$; $S_k = E[\omega(k)\upsilon^T(k)]$; $R_k = E[\upsilon(k)\upsilon^T(k)]$.

The estimation error variance matrix $P(k+1|k)$ can be calculated by the recurrence equation:

$$\begin{aligned} P(k+1|k) &= E[\tilde{x}(k+1|k)\tilde{x}^T(k+1|k)] \\ &= E[\{[A-K(k)C]\tilde{x}(k|k-1) + G\omega_k + K(k)v(k)\} \\ &\quad \times \{[A-K(k)C]\tilde{x}(k|k-1) + G\omega_k + K(k)v(k)\}^T] \\ &= E[A-K(k)C]P(k|k-1)[A-K(k)C]^T + K(k)R_kK^T(k) \\ &\quad + GQ_kF^T - GS_kK^T(k) - K(k)S_k^TG^T \\ &= AP(k|k-1)A^T - [AP(k|k-1)C^T + GS_k] \\ &\quad \times [CP(k|k-1)C^T + R_k]^{-1} \\ &\quad \times [CP(k|k-1)A^T + S_k^TG^T] + GQ_kG^T, \end{aligned} \tag{13}$$

where $Q_k = E[\omega(k)\omega^T(k)]$.

Therefore the calculation procedures with the optimal state estimator are defined as:

First, specify the initial conditions $x(0)$ and $P(0)$; then, respectively, calculate $K(0)$ with Eq. 12, $\hat{x}(1|0)$ with Eq. 9, $P(1|0)$ with Eq. 13, $K(1)$ with the $P(1|0)$ and (12), $\hat{x}(2|1)$ with $K(1)$.

Then repeat the steps above, so the state variable $\hat{x}(k+1|k)$ at any time can be predicted.

In other words, the MPC algorithm with a state estimator is to use the measurable sequence $y(0), y(1) \ldots y(k)$ to calculate the linear optimal estimation $\hat{x}(k+1|k)$ so that the estimation error variance matrix $P(k|k-1)$ of the estimation error $\tilde{x}(k|k-1) = x(k|k-1) - \hat{x}(k|k-1)$ can be minimized.

5 Simulation analysis

In this section, a certain supply vessel will be used as the simulation object. The main parameters of the vessel are given in Table 1.

The bis-scaled system mass matrix M and damping matrix D for the supply vessel in [23]:

$$M'' = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix},$$

Table 1 Parameters of the supply vessel

Ship parameters	Value
Length overall	76.2 m
Beam	18.8 m
Depth	8.25 m
Draught	6.25 m
Mass	4200 t
Engine power	3533 kW

$$D'' = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix}.$$

The simulation parameters are set as follows: original position and heading of vessel $\eta = [0 \quad 0 \quad 0]$, the desired position and heading $r = [10 \quad 10 \quad 30°]$, sampling time is 0.5 s, simulation time 1000 s; the force constraints of surge and sway are ±5000 KN, the moment constraint of yaw is ±10,000 KN m, the position and heading constraints are ±20 m and ±45°; prediction horizon $p = 100$, control horizon $m = 5$. Then, respectively, simulation on MPC controller with state estimator (has no constraint), MPC controller without state estimator (has no constraint), MPC controller with state estimator (has constraints), and MPC controller without state estimator (has constraints). The simulation results are shown in Figs. 3, 4, 5, and 6.

For easy visualization, the simulation times of Figs. 3, 4, 5, and 6 are held only within 300, 100, 300 and 300 s. As can be seen from the output position results:

1. When without constraints, the MPC controller with the state estimator can produce larger forces and torque, so it can track the desired position in relatively short period of time than that without the state estimator (see Figs. 3, 4);
2. When with constraints, the MPC controller with the state estimator can also track the desired position

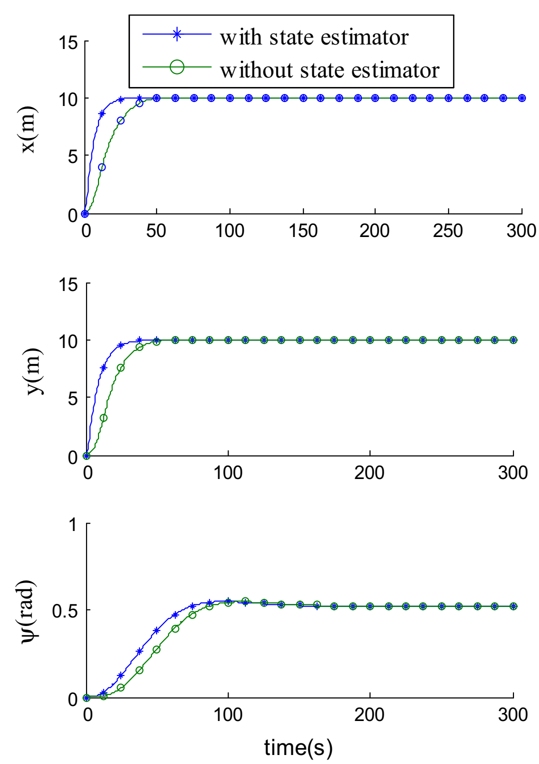

Fig. 3 The position and heading output of the dynamic positioning ship without constraint

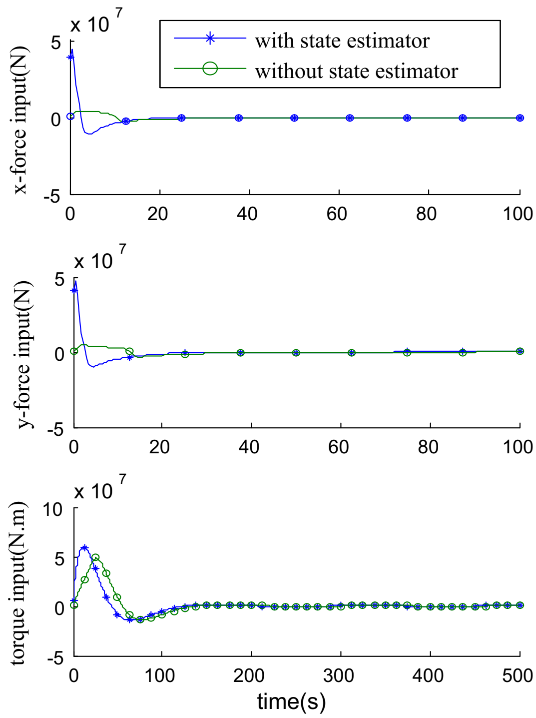

Fig. 4 The control input of the dynamic positioning ship without constraint

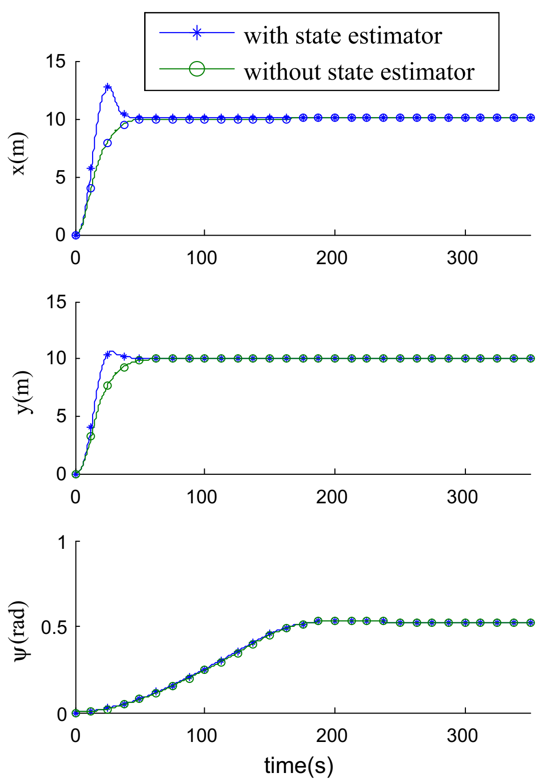

Fig. 5 The position and heading output of the dynamic positioning ship with constraint

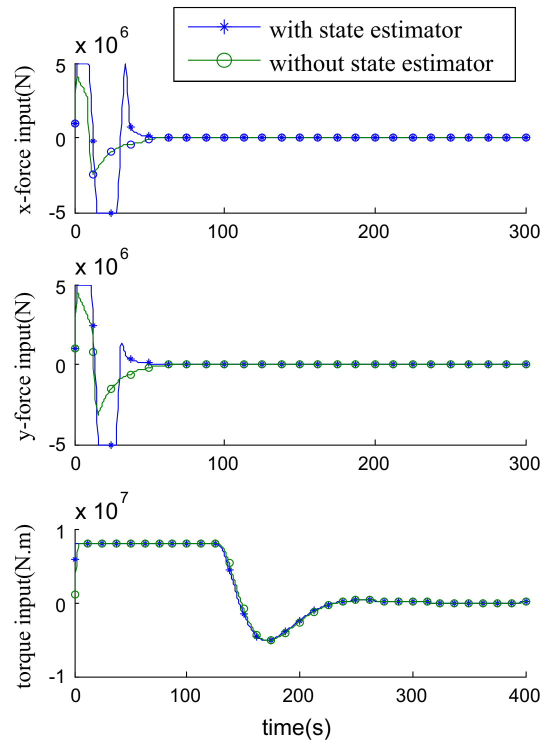

Fig. 6 The control input of the dynamic positioning ship with constraint

smoothly and quickly, but it produces some overshoots in the *x*- and *y*-direction, and the fluctuations of the forces are greater (see Figs. 5, 6);

3. In terms of the control forces and torque: at the primary stage because of the difference between the initial position and desired position is too much, therefore the primary stage requires a lot of forces and torque to reduce the difference. But when the output location is equal to the desired location, then the control forces and torques will be steady, this steady state process costs a short time.

Therefore, in terms of the simulation results, the MPC controller with the state estimator is effective and can achieve the fast dynamic response and has a good robustness. The state estimator can meet the MPC controller with or without constraints at the same time, and can achieve the good control quality.

6 Conclusions

In this paper, the MPC algorithm with a state estimator is proposed. In addition, based on the algorithm, a DP controller is designed to solve the problem when not all the states of the system can be measured. The simulation results show that the controller with the state estimator has the characteristics of fast, stable, and accurate responses. At the same time, the results verify the model predictive controller with the state estimator can very close to the control performance of the dynamic positioning system without state estimator. It is scheduled to expand applicability of the MPC algorithm by the platform supply vessel model (scale of 1:50) with two main propellers and one bow tunnel thruster in the tank in future.

Acknowledgements This work was supported by the Fundamental Research Funds for the Central Universities (3132014332, 3132015025) and Natural Science Foundation of Liaoning Province (2015020132).

References

1. Balchen JG, Jenssen NA, Mathisen E et al (1980) A dynamic positioning system based on Kalman filtering and optimal control. Model Identif Control 1(3):135–163
2. Grimble MJ, Patton RJ, Wise DA (1980) The design of dynamic ship positioning control systems using stochastic optimal control theory. Optim Control Appl Methods 1(2):167–202
3. Fossen TI, Grovlen A (1998) Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping. IEEE Trans Control Syst Technol 6(1):121–128
4. Ngo KB, Mahony R, Jiang ZP (2005) Integrator backstepping using barrier functions for systems with multiple state constraints. In: 44th IEEE conference on decision and control, 2005 and 2005 European control conference. CDC-ECC'05. IEEE, 2005, pp 8306–8312
5. Yamamoto I, Matsuura M, Yamaguchi Y et al (1997) Dynamic positioning system based on nonlinear programming for offshore platforms. In: Proceedings of the international offshore and polar engineering conference, vol 3, pp 632–640
6. Daigo K, Yamamoto I, Matsuura M et al (1998) Dynamic positioning system for FPSOs with fuzzy control logic based on a nonlinear programming algorithm. In: Proceedings of the international conference on offshore mechanics and arctic engineering, OMAE, 11 pp
7. Fossen TI, Strand JP (1998) Passive nonlinear observer design for ships using Lyapunov methods: full-scale experiments with a supply vessel. Automatica 35(1):3–16
8. Muhammad S, Dòria-Cerezo A (2012) Passivity-based control applied to the dynamic positioning of ships. IET Control Theory Appl 6(5):680–688
9. Yamamoto I, Matsuura M, Hirayama H et al (2001) Dynamic positioning system of offshore platform by advanced control. In: Proceedings of the international conference on offshore mechanics and arctic engineering, OMAE, vol 1, pp 1025–1033
10. Kim MH, Inman DJ (2001) Application of a sliding mode observer for dynamic positioning of ships. In: Proceedings of the ASME design engineering technical conference, Pittsburgh, Pennsylvania
11. Xie W, Fu M, Shi X (2013) Adaptive sliding-mode passive observer design for dynamic positioning vessel. Control Theory Appl 30(1):131–136
12. Du J, Yang Y, Guo C et al (2013) Output feedback control for dynamic positioning system of a ship based on a high gain observer. Control Theory Appl 30(11):486–491
13. Du J, Hu X, Liu H et al (2015) Adaptive robust output feedback control for a marine dynamic positioning system based on a high-gain observer. IEEE Trans Neural Netw Learn Syst 26(11):2775–2786
14. Yamamoto I, Terada Y (1999) Development of inverse LMI dynamic positioning system of offshore platform by advanced control. In: IEEE conference on control applications—proceedings, vol 2, pp 1118–1122
15. Katebi MR, Grimble MJ, Zhang Y (1997) H∞ robust control design for dynamic ship positioning. IEE Proc Control Theory Appl 144(2):110–120
16. Peng Z, Wang D, Hu X (2011) Robust adaptive formation control of underactuated autonomous surface vehicles with uncertain dynamics. IET Control Theory Appl 5(12):1378–1387
17. Katebi MR, Yamamoto I, Matsuura M et al (2001) Robust dynamic ship positioning control system design and applications. Int J Robust Nonlinear Control 11(13):1257–1284
18. Wang YH (2006) Application of model predictive control to dynamic positioning system. Master thesis, Harbin Engineering University, Harbin
19. Guerreiro BJ, Silvestre C, Cunha R et al (2014) Trajectory tracking nonlinear model predictive control for autonomous surface craft. IEEE Trans Control Syst Technol 22(6):2160–2175
20. Wang Y, Shi X, Bian X (2007) Restriction control of marine dynamic positioning system based on model predictive control. Ship Eng 3:22–25
21. Fossen TI (1994) Guidance and control of ocean vehicles. Wiley, New York, pp 21–25
22. Maciejowski JM (2002) Predictive control: with constraints. Pearson education, New Jersey, pp 83–117
23. Fossen TI, Sagatun SI, Sφrensen AJ (1996) Identification of dynamically positioned ships. Control Eng tice 4(3):369–376